2024 AMC 12A Problems

Problem 1

What is the value of

$$9901 \cdot 101 - 99 \cdot 10101?$$

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 200 \qquad \textbf{(D)}\ 202 \qquad \textbf{(E)}\ 2020$

Problem 2

A model used to estimate the time it will take to hike to the top of the mountain on a trail is of the form $T = aL + bG,$ where a and b are constants, T is the time in minutes, L is the length of the trail in miles, and G is the altitude gain in feet. The model estimates that it will take 69 minutes to hike to the top if a trail is 1.5 miles long and ascends 800 feet, as well as if a trail is 1.2 miles long and ascends 1100 feet. How many minutes does the model estimate it will take to hike to the top if the trail is 4.2 miles long and ascends 4000 feet?

$\textbf{(A)}\ 240 \qquad \textbf{(B)}\ 246 \qquad \textbf{(C)}\ 252 \qquad \textbf{(D)}\ 258 \qquad \textbf{(E)}\ 264$

Problem 3

The number 2024 is written as the sum of not necessarily distinct two-digit numbers. What is the least number of two-digit numbers needed to write this sum?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 22 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 24$

Problem 4

What is the least value of n such that $n!$ is a multiple of 2024?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 22 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 253$

Problem 5

A data set containing 20 numbers, some of which are 6, has mean 45. When all the 6s are removed, the data set has mean 66. How many 6s were in the original data set?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Problem 6

The product of three integers is 60. What is the least possible positive sum of the three integers?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 13$

Problem 7

In $\triangle ABC$, $\angle ABC = 90^\circ$ and $BA = BC = \sqrt{2}$. Points $P_1, P_2, \dots, P_{2024}$ lie on hypotenuse $\overline{AC}$ so that $AP_1 = P_1P_2 = P_2P_3 = \cdots = P_{2023}P_{2024} = P_{2024}C$. What is the length of the vector sum

$$\overrightarrow{BP_1} + \overrightarrow{BP_2} + \overrightarrow{BP_3} + \cdots + \overrightarrow{BP_{2024}}?$$

$\textbf{(A) } 1011 \qquad \textbf{(B) } 1012 \qquad \textbf{(C) } 2023 \qquad \textbf{(D) } 2024 \qquad \textbf{(E) } 2025$

Problem 8

How many angles θ with $0 \le \theta \le 2\pi$ satisfy

$$\log(\sin(3\theta)) + \log(\cos(2\theta)) = 0?$$

$\textbf{(A) } 0 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 3 \qquad \textbf{(E) } 4$

Problem 9

Let M be the greatest integer such that both $M + 1213$ and $M + 3773$ are perfect squares. What is the units digit of M?

$\textbf{(A) } 1 \qquad \textbf{(B) } 2 \qquad \textbf{(C) } 3 \qquad \textbf{(D) } 6 \qquad \textbf{(E) } 8$

Problem 10

Let α be the radian measure of the smallest angle in a $3{-}4{-}5$ right triangle. Let β be the radian measure of the smallest angle in a $7{-}24{-}25$ right triangle. In terms of α, what is β?

$\textbf{(A) } \frac{\alpha}{3} \qquad \textbf{(B) } \alpha - \frac{\pi}{8} \qquad \textbf{(C) } \frac{\pi}{2} - 2\alpha \qquad \textbf{(D) } \frac{\alpha}{2} \qquad \textbf{(E) } \pi - 4\alpha$

Problem 11

There are exactly K positive integers b with $5 \leq b \leq 2024$ such that the base-b integer 2024_b is divisible by 16 (where 16 is in base ten). What is the sum of the digits of K?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 20 **(E)** 21

Problem 12

The first three terms of a geometric sequence are the integers $a,\ 720,$ and $b,$ where $a < 720 < b.$ What is the sum of the digits of the least possible value of b?

(A) 9 **(B)** 12 **(C)** 16 **(D)** 18 **(E)** 21

Problem 13

The graph of

$$y = e^{x+1} + e^{-x} - 2$$

has an axis of symmetry. What is the reflection of the point $(-1, \frac{1}{2})$ over this axis?

(A) $\left(-1, -\dfrac{3}{2}\right)$ **(B)** $(-1, 0)$ **(C)** $(-1, \frac{1}{2})$ **(D)** $\left(0, \dfrac{1}{2}\right)$ **(E)** $\left(3, \dfrac{1}{2}\right)$

Problem 14

The numbers, in order, of each row and the numbers, in order, of each column of a 5×5 array of integers form an arithmetic progression of length $5.$ The numbers in positions $(5,5)$, $(2,4)$, $(4,3)$, and $(3,1)$ are $0, 48, 16,$ and $12,$ respectively. What number is in position $(1,2)$?

$$\begin{bmatrix} . & ? & . & . & . \\ . & . & . & 48 & . \\ 12 & . & . & . & . \\ . & . & 16 & . & . \\ . & . & . & . & 0 \end{bmatrix}$$

$\textbf{(A)}\ 19 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 29 \qquad \textbf{(D)}\ 34 \qquad \textbf{(E)}\ 39$

Problem 15

The roots of $x^3 + 2x^2 - x + 3$ are $p, q,$ and r. What is the value of

$$(p^2 + 4)(q^2 + 4)(r^2 + 4)?$$

$\textbf{(A)}\ 64 \qquad \textbf{(B)}\ 75 \qquad \textbf{(C)}\ 100 \qquad \textbf{(D)}\ 125 \qquad \textbf{(E)}\ 144$

Problem 16

A set of 12 tokens ---- 3 red, 2 white, 1 blue, and 6 black ---- is to be distributed at random to 3 game players, 4 tokens per player. The probability that some player gets all the red tokens, another gets all the white tokens, and the remaining player gets the blue token can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

$\textbf{(A)}\ 387 \qquad \textbf{(B)}\ 388 \qquad \textbf{(C)}\ 389 \qquad \textbf{(D)}\ 390 \qquad \textbf{(E)}\ 391$

Problem 17

The first three terms of a geometric sequence are the integers $a,$ $720,$ and b, where $a < 720 < b.$ What is the sum of the digits of the least possible value of b?

$\textbf{(A) } 9 \qquad \textbf{(B) } 12 \qquad \textbf{(C) } 16 \qquad \textbf{(D) } 18 \qquad \textbf{(E) } 21$

Problem 18

On top of a rectangular card with sides of length 1 and $2+\sqrt{3}$, an identical card is placed so that two of their diagonals line up, as shown ($\overline{AC}$, in this case).

Continue the process, adding a third card to the second, and so on, lining up successive diagonals after rotating clockwise. In total, how many cards must be used until a vertex of a new card lands exactly on the vertex labeled B in the figure?

$\textbf{(A) } 6 \qquad \textbf{(B) } 8 \qquad \textbf{(C) } 10 \qquad \textbf{(D) } 12 \qquad \textbf{(E) } \text{No new vertex will land on } B.$

Problem 19

Cyclic quadrilateral $ABCD$ has lengths

$$BC = CD = 3\text{and } DA = 5$$

with $\angle CDA = 120^\circ$.

What is the length of the shorter diagonal of $ABCD$?

$\textbf{(A) } \frac{31}{7} \qquad \textbf{(B) } \frac{33}{7} \qquad \textbf{(C) } 5 \qquad \textbf{(D) } \frac{39}{7} \qquad \textbf{(E) } \frac{41}{7}$

Problem 20

Points P and Q are chosen uniformly and independently at random on sides $\overline{AB}$ and $\overline{AC}$, respectively, of equilateral triangle $\triangle ABC$. Which of the following intervals contains the probability that the area of $\triangle APQ$ is less than half the area of $\triangle ABC$?

(A) $\left[\frac{3}{8},\frac{1}{2}\right]$ **(B)** $\left(\frac{1}{2},\frac{2}{3}\right]$ **(C)** $\left(\frac{2}{3},\frac{3}{4}\right]$ **(D)** $\left(\frac{3}{4},\frac{7}{8}\right]$ **(E)** $\left(\frac{7}{8},1\right]$

Problem 21

Suppose that $a_1 = 2$ and the sequence (a_n) satisfies the recurrence relation

$$\frac{a_n - 1}{n-1} = \frac{a_{n-1}+1}{(n-1)+1}$$

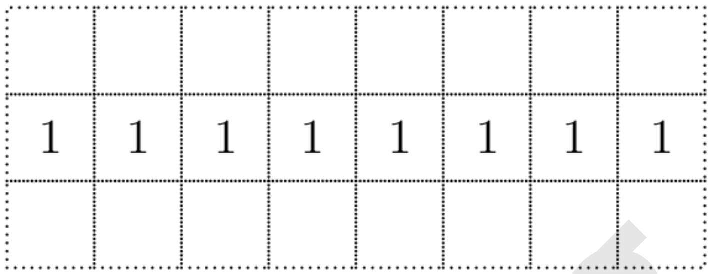
for all $n \geq 2$. What is the greatest integer less than or equal to

$$\sum_{n=1}^{100} a_n^2?$$

(A) 338,550 **(B)** 338,551 **(C)** 338,552 **(D)** 338,553 **(E)** 338,554

Problem 22

The figure below shows a dotted grid 8 cells wide and 3 cells tall consisting of $1'' \times 1''$ squares. Carl places 1-inch toothpicks along some of the sides of the squares to create a closed loop that does not intersect itself. The numbers in the cells indicate the number of sides of that square that are to be covered by toothpicks, and any number of toothpicks are allowed if no number is written. In how many ways can Carl place the toothpicks?



(A) 130 **(B)** 144 **(C)** 146 **(D)** 162 **(E)** 196

Problem 23

What is the value of

$$\tan^2\frac{\pi}{16}\cdot\tan^2\frac{3\pi}{16}+\tan^2\frac{\pi}{16}\cdot\tan^2\frac{5\pi}{16}+\tan^2\frac{3\pi}{16}\cdot\tan^2\frac{7\pi}{16}+\tan^2\frac{5\pi}{16}\cdot\tan^2\frac{7\pi}{16}?$$

(A) 28 **(B)** 68 **(C)** 70 **(D)** 72 **(E)** 84

Problem 24

A *disphenoid* is a tetrahedron whose triangular faces are congruent to one another. What is the least total surface area of a disphenoid whose faces are scalene triangles with integer side lengths?

(A) $\sqrt{3}$ **(B)** $3\sqrt{15}$ **(C)** 15 **(D)** $15\sqrt{7}$ **(E)** $24\sqrt{6}$

Problem 25

A graph is *symmetric* about a line if the graph remains unchanged after reflection in that line. For how many quadruples of integers (a,b,c,d), where $|a|,|b|,|c|,|d| \le 5$ and c and d are not both 0, is the graph of

$$y = \frac{ax+b}{cx+d}$$

symmetric about the line $y = x$?

$\textbf{(A)}\ 1282 \qquad \textbf{(B)}\ 1292 \qquad \textbf{(C)}\ 1310 \qquad \textbf{(D)}\ 1320 \qquad \textbf{(E)}\ 1330$