2025 AMC 10B Problems

Problem 1

The instructions on a 350-gram bag of coffee beans say that proper brewing of a large mug of pour-over coffee requires 20 grams of coffee beans. What is the greatest number of properly brewed large mugs of coffee that can be made from the coffee beans in that bag?

(A) 16 (B) 17 (C) 18 (D) 19 (E) 20

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Problem 2

Jerry wrote down the ones digit of each of the first 2025 positive squares:1,4,9,6,5,6, …. What is the sum of all the numbers Jerry wrote down?

(A) 9025 (B) 9070 (C) 9090 (D) 9115 (E) 9160

Problem 3

A Pascal-like triangle has 10 as the top row and 10 followed by 1 as the second row. In each subsequent row the first number is 10, the last number is 1, and, as in the standard Pascal Triangle, each other number in the row is the sum of the two numbers directly above it. The first four rows are shown below.

10

10 1

10 11 1

10 21 12 1

What is the sum of the digits of the sum of the numbers in the 11th row?

(A) 11 (B) 13 (C) 14 (D) 16 (E) 17

Problem 4

The value of the two-digit number $a\ b$ in base seven equals the value of the two-digit number $b\ a$ in base nine. What is $a + b$?

(A) 7 (B) 9 (C) 10 (D) 11 (E) 14

Problem 5

In $\triangle ABC$, $AB = 10$, $AC = 18$, and $\angle B = 130°$. Let O be the center of the circle containing points A, B, and C. What is the degree measure of $\angle CAO$?

(A) 20 (B) 30 (C) 40 (D) 50 (E) 60

Problem 6

The line $y = \frac{1}{3}x + 1$ divides the square region defined by $0 \le x \le 2$ and $0 \le y \le 2$ into an upper and a lower region. The line $x = a$ divides the lower region

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into two regions of equal area. Then a can be written as $\sqrt{s}-t$ where s and t are positive integers. What is $s+t$?

(A) 18 (B) 19 (C) 20 (D) 21 (E) 22

Problem 7

Frances stands 15 meters directly south of a locked gate in a fence that runs east-west. Immediately behind the fence is a box of chocolates, located x meters east of the locked gate. An unlocked gate lies 9 meters east of the box, and another unlocked gate lies 8 meters west of the locked gate. Frances can reach the box by walking toward an unlocked gate, passing through it, and walking toward the box. It happens that the total distance Frances would travel is the same via either unlocked gate. What is the value of x ?

(A) $3\frac{2}{3}$ (B) $3\frac{3}{7}$ (C) $3\frac{4}{7}$ (D) $3\frac{5}{7}$ (E) $3\frac{6}{7}$

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Problem 8

Emmy says to Max, "I ordered 36 math club sweatshirts today." Max asks, "How much did each shirt cost?" Emmy responds, "I'll give you a hint. The total cost was \$$ABB.BA$, where A and B are digits and $A \neq 0$." After a pause, Max says, "That was a good price." What is $A+B$?

(A) 7 (B) 8 (C) 11 (D) 14 (E) 15

Problem 9

How many ordered triples of integers (x, y, z) satisfy the following system of inequalities?

$$-x - y - z \leqslant -2$$

$$-x + y + z \leqslant 2$$

$$x - y + z \leqslant 2$$

$$x + y - z \leqslant 2$$

(A) 4 (B) 8 (C) 11 (D) 15 (E) 17

Problem 10

Let $f(n) = n^3 - 5n^2 + 2n + 8$, and let $g(n) = n^3 - 6n^2 + 5n + 12$. What is the sum of all integers n such that $\frac{f(n)}{g(n)}$ is also an integer?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Problem 11

On Monday, 6 students went to the tutoring center at the same time, and each one was randomly assigned to one of the 6 tutors on duty. On Tuesday, the same 6 students showed up, the same 6 tutors were on duty, and the students were again randomly assigned to the tutors. What is the probability that exactly 2 students met with the same tutor both Monday and Tuesday?

(A) $\frac{1}{16}$ (B) $\frac{3}{16}$ (C) $\frac{1}{4}$ (D) $\frac{3}{8}$ (E) $\frac{1}{2}$

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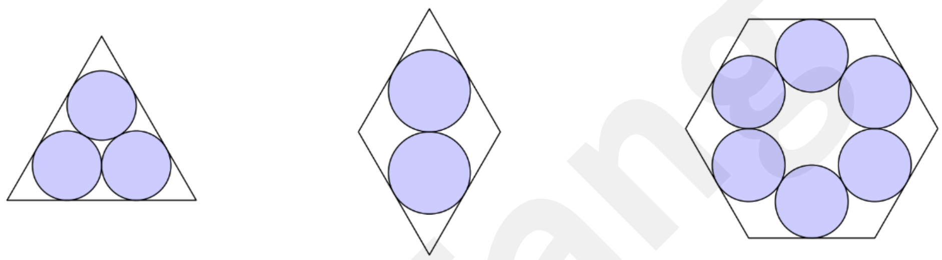
Problem 12

The figure below shows an equilateral triangle, a rhombus with a 60° angle, and a regular hexagon, each of them containing some mutually tangent congruent disks. Let T, R, and H, respectively, denote the ratio in each case of the total area of the disks to the area of the enclosing polygon. Which of the following is true?



(A) $T = R = H$ (B) $H < R = T$ (C) $H = R < T$ (D) $H < R < T$ (E) $H < T < R$

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Problem 13

The altitude to the hypotenuse of a $30^\circ - 60^\circ - 90^\circ$ right triangle is divided into two segments of lengths $x < y$ by the median to the shortest side of the triangle. What is the ratio $\frac{x}{x+y}$?

(A) $\frac{3}{7}$ (B) $\frac{\sqrt{3}}{4}$ (C) $\frac{4}{9}$ (D) $\frac{5}{11}$ (E) $\frac{4\sqrt{3}}{15}$

Problem 14

Nine athletes, no two of whom are the same height, try out for the basketball team. One at a time, they draw a wristband at random, without replacement, from a bag containing 3 blue bands, 3 red bands, and 3 green bands. They are divided into a blue group, a red group, and a green group. The tallest member of each group is named the group captain. What is the probability that the group captains are the three tallest athletes?

(A) $\frac{2}{9}$ (B) $\frac{2}{7}$ (C) $\frac{9}{28}$ (D) $\frac{1}{3}$ (E) $\frac{3}{8}$

Problem 15

The sum

$$\sum_{k=1}^{\infty} \frac{1}{k^3 + 6k^2 + 8k}$$

can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a + b$?

(A) 89 (B) 97 (C) 102 (D) 107 (E) 129

Problem 16

A circle has been divided into 6 sectors of 6 different sizes. Then 2 of the sectors are painted red, 2 painted green, and 2 painted blue so that no two neighboring sectors are painted the same color. One such coloring is shown below. How many different colorings are possible?

(A) 12 (B) 16 (C) 18 (D) 24 (E) 28

Problem 17

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Consider a decreasing sequence of n positive integers $x_1 > x_2 > \cdots > x_n$ that satisfies the following conditions:

- The average of the first 3 terms in the sequence is 2025.
- For all $4 \le k \le n$, the average of the first k terms is 1 less than the average of the first k - 1 terms.

What is the greatest possible value of n?

(A) 1013 (B) 1014 (C) 1016 (D) 2016 (E) 2025

Problem 18

What is the ones digit of the sum $\lfloor\sqrt{1}\rfloor + \lfloor\sqrt{2}\rfloor + \lfloor\sqrt{3}\rfloor + \cdots + \lfloor\sqrt{2025}\rfloor$?

(Recall that $\lfloor x \rfloor$ represents the greatest integer less than or equal to x.)

(A) 1 (B) 2 (C) 3 (D) 5 (E) 8

Problem 19

A container has a 1 × 1 square bottom, a 3 × 3 open square top, and four congruent trapezoidal sides, as shown. Starting when the container is empty, a hose that runs water at a constant rate takes 35 minutes to fill the container up to the midline of the trapezoids. How many more minutes will it take to fill the remainder of the container?

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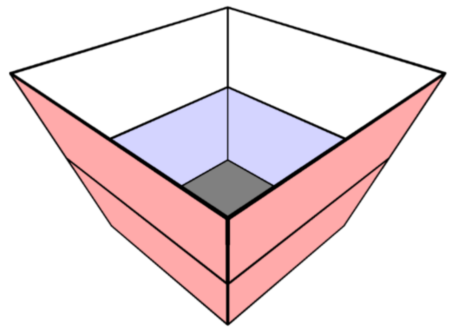

(A) 70 (B) 85 (C) 90 (D) 95 (E) 105

Problem 20

Four congruent semicircles are inscribed in a square of side length 1 so that their diameters are on the sides of the square, one endpoint of each diameter is at a vertex of the square, and adjacent semicircles are tangent to each other. A small circle centered at the center of the square is tangent to each of the four semicircles, as shown below.

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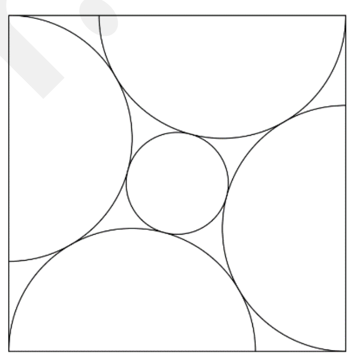

The diameter of the small circle can be written as $(\sqrt{a} + b)(\sqrt{c} + d)$, where a, b, c, and d are integers. What is $a + b + c + d$?

(A) 3 (B) 5 (C) 8 (D) 9 (E) 11

Problem 21

Each of the 9 squares in a 3×3 grid is to be colored red, blue, or yellow in such a way that each red square shares an edge with at least one blue square, each blue square shares an edge with at least one yellow square, and each yellow square shares an edge with at least one red square. Colorings that can be obtained from one another by rotations and/or reflections are to be considered the same. How many different colorings are possible?

(A) 3 (B) 9 (C) 12 (D) 18 (E) 27

Problem 22

A seven-digit positive integer is chosen at random. What is the probability that the number is divisible by 11, given that the sum of its digits is 61?

(A) $\frac{3}{14}$ (B) $\frac{3}{11}$ (C) $\frac{2}{7}$ (D) $\frac{4}{11}$ (E) $\frac{3}{7}$

Problem 23

A rectangular grid of squares has 141 rows and 91 columns. Each square has room for two numbers. Horace and Vera each fill in the grid by putting the numbers from 1 through $141 \times 91 = 12{,}831$ into the squares. Horace fills the grid horizontally: he puts 1 through 91 in order from left to right into row 1, puts 92 through 182 into row 2 in order from left to right, and continues similarly through row 141. Vera fills the grid vertically: she puts 1 through 141 in order from top to bottom into column 1, then 142 through 282 into

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column 2 in order from top to bottom, and continues similarly through column 91. How many squares get two copies of the same number?

(A) 7 (B) 10 (C) 11 (D) 12 (E) 19

Problem 24

A frog hops along the number line according to the following rules.

- It starts at 0.
- If it is at 0, then it moves to 1 with probability $\frac{1}{2}$ and it disappears with probability $\frac{1}{2}$.
- For n = 1, 2 or 3, if it is at n, then it moves to $n + 1$ with probability $\frac{1}{4}$, it moves to n -1 with probability $\frac{1}{4}$, and it disappears with probability $\frac{1}{2}$.

What is the probability that the frog reaches 4?

(A) $\frac{1}{101}$ (B) $\frac{1}{100}$ (C) $\frac{1}{99}$ (D) $\frac{1}{98}$ (E) $\frac{1}{97}$

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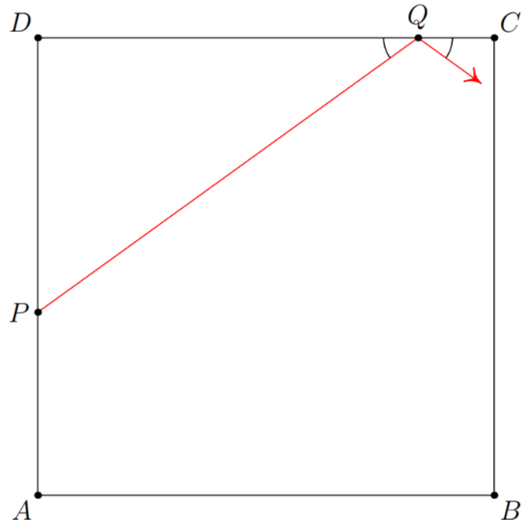
Problem 25

Square $ABCD$ has sides of length 4. Points P and Q lie on $\overline{AD}$ and $\overline{CD}$, respectively, with $AP = \frac{8}{5}$ and $DQ = \frac{10}{3}$. A path begins along the segment from P to Q and continues by reflecting against the sides of $ABCD$ (with congruent incoming and outgoing angles). If the path hits a vertex of the square, it terminates there; otherwise, it continues forever. At which vertex does the path terminate?



(A) A (B) B (C) C (D) D (E) The path continues forever.

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